
Algorithm 1 Reduction from 3-SAT to SUBSETSUM

Require: A 3-CNF formula ϕ with variables x_1, \dots, x_n and clauses C_1, \dots, C_m

Ensure: A SUBSETSUM instance (S, T)

- 1: Let each number have $n + m$ digit columns.
- 2: The first n columns are variable columns.
- 3: The last m columns are clause columns.
- 4: Initialize $S \leftarrow \emptyset$.
- 5: **for** $i = 1$ to n **do**
- 6: Create two $(n + m)$ -digit numbers v_i^t and v_i^f , initially all 0's.
- 7: Set the i th variable digit of both v_i^t and v_i^f to 1.
- 8: **for** $j = 1$ to m **do**
- 9: **if** x_i appears in clause C_j **then**
- 10: Set the j th clause digit of v_i^t to 1.
- 11: **end if**
- 12: **if** $\neg x_i$ appears in clause C_j **then**
- 13: Set the j th clause digit of v_i^f to 1.
- 14: **end if**
- 15: **end for**
- 16: Add v_i^t and v_i^f to S .
- 17: **end for**
- 18: **for** $j = 1$ to m **do**
- 19: Create two $(n + m)$ -digit slack numbers $s_{j,1}$ and $s_{j,2}$, initially all 0's.
- 20: Set the j th clause digit of $s_{j,1}$ to 1.
- 21: Set the j th clause digit of $s_{j,2}$ to 2.
- 22: Add $s_{j,1}$ and $s_{j,2}$ to S .
- 23: **end for**
- 24: Define the target number

$$T = \underbrace{11 \cdots 1}_{n \text{ variable digits}} \underbrace{44 \cdots 4}_{m \text{ clause digits}} .$$

25: **return** (S, T) .
